

1. We're interested in the convergence of the series  $\sum_{n=1}^{\infty} \frac{n^2+3n^5+8n!}{n^n}$ .

Define  $a(n) = \frac{n^2+3n^5+8n!}{n^n}$ . Find the **numerical** values of  $\frac{a(n+1)}{a(n)}$  for large values of  $n$  (say 1000, 10000, and 100000). Using Mathematica, calculate the actual limit. What does the ratio test say about the series?

2. The MaClaurin series for  $\sin(x)$  is

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^k \frac{x^{2n+1}}{(2n+1)!} = x - \frac{x^3}{6} + \frac{x^5}{120} + \dots$$

By truncating the series at some point, we get a Taylor polynomial. Plot the degree 4 Taylor polynomial of  $\sin(x)$  and the sine function on the same graph, in the interval  $[-8, 8]$ . Do the same for degrees 8, 12, and 16. For roughly what values of  $x$  is the Taylor polynomial a good approximation to the sine function for each of these degrees?